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Letter to the Editor

Flow velocity profiles in rectangular channels containing two liquid layers of different densities and viscosities for sedimentation field flow fractionation

Sir,

A numerical method for calculation of the flow velocity profiles in channels containing several liquid layers of different densities and viscosities has recently been presented by Janča[1]. Unfortunately, both the approach to the problem and the results seem incorrect. The failure of the proposed solution is manifested in the discontinuous velocity profiles at the interface between two moving liquids of different viscosities. This would mean no viscosity or no momentum transport at the interface.

To clarify the problem, let us consider a rectangular channel possessing a sufficiently high aspect ratio, $A = w/b$, w being the channel width and b its thickness. The lower part of the channel between 0 and x_0 is occupied by a heavier liquid (viscosity, η_1), the upper part between x_0 and b by a lighter one (viscosity, η_2).

Using the one-dimensional approximation, commonly used in field flow fractionation (FFF), we may write the steady flow equations

$$\frac{d^2u_i}{dx^2} = -\frac{\Delta P}{\eta_i L} = -K_i \qquad i = 1, 2 \tag{1}$$

where u_i and η_i are the fluid velocity and viscosity, respectively, and, ΔP is the pressure drop across the channel of length L.

The boundary problem can be formulated as follows:

$$u_1(0) = u_2(b) = 0 \tag{2}$$

$$u_1(x_0) = u_2(x_0) \tag{3}$$

$$\eta_1 \cdot \frac{du_1(x_0)}{dx} = \eta_2 \cdot \frac{du_2(x_0)}{dx} \tag{4}$$

Conditions 2 are classical in problems dealing with the laminar flow. Eqn. 4 expresses the balance of friction forces acting at the interface.

By solving the system 1–4, we obtain

$$u_1 = \frac{K_1}{2} \cdot x(x_0 - x) + \frac{K_2 x}{2\,x_0}(b - x_0)[(b + x_0) - f(\eta)] \tag{5}$$

and

$$u_2 = \frac{K_2}{2}(b - x)[(b + x) - f(\eta)] \tag{6}$$

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where

$$f(\eta) = \frac{\eta_1(b^2 - x_0^2) + \eta_2 x_0^2}{\eta_1(b - x_0) + \eta_2 x_0}$$

The velocity profiles 5 and 6 do not display any discontinuity; the flow of the more dense liquid is influenced by the flow of the less dense one, and *vice versa*.

Denoting the volumetric flow-rates of the liquids in the channel as q_i, we may write

$$q_1 = w \int_0^{x_0} u_1 \mathrm{d}x \qquad q_2 = w \int_{x_0}^{b} u_2 \mathrm{d}x$$

or

$$\frac{q_1}{w} = \frac{K_1}{12} \cdot x_0^3 + \frac{K_2}{4} \cdot x_0(b - x_0)[(b + x_0) - f(\eta)] \tag{7}$$

and

$$\frac{q_2}{w} = \frac{K_2}{2} (b - x_0)\left[b^2 - \frac{b^3 - x_0^3}{3(b - x_0)} - \frac{b - x_0}{2} \cdot f(\eta)\right] \tag{8}$$

Given q_1 or q_2, the interface position, x_0, can be calculated from eqn. 7 or 8, and the corresponding velocity profiles are determined by eqn. 5 and 6.

If merely the less dense liquid is pumped into the channel ($q_1 = 0$), then, according to eqn. 7, $x_0 = 0$ and from eqn. 8 we arrive at the familiar formula:

$$\frac{q^2}{wb} = \frac{K_2}{12} \cdot b^2$$

Similarly, if $q_2 = 0$, then $x_0 = b$ and

$$\frac{q_1}{wb} = \frac{K_1}{12} \cdot b^2$$

If both liquids display the same viscosity, then $K = K_1 = K_2$, $f(\eta) = b$ and

$$u_1 = u_2 = \frac{K}{2} \cdot x(b - x)$$

as expected.

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1 J. Janča, *J. Chromatogr.*, 404 (1987) 23–32.

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